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Paper-IIIB

Topic: Heisenberg Uncertainty Principle and its significance

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Heisenberg Uncertainty Principle and its significance

The uncertainty principle is certainly one of the most famous and important aspects of quantum mechanics. It has often been regarded as the most distinctive feature in which quantum mechanics differs from classical theories of the physical world. Roughly speaking, the uncertainty principle (for position and momentum) states that one cannot assign exact simultaneous values to the position and momentum of a physical system. Rather, these quantities can only be determined with some characteristic 'uncertainties' that cannot become arbitrarily small simultaneously. But what is the exact meaning of this principle, and indeed, is it really a principle of quantum mechanics? (In his original work, Heisenberg only speaks of uncertainty relations.) And, in particular, what does it mean to say that a quantity is determined only up to some uncertainty? These are the main questions we will explore in the following, focusssing on the views of Heisenberg.

In the field of quantum mechanics, Heisenberg's uncertainty principle is a fundamental theory that explains why it is impossible to measure more than one quantum variables simultaneously. Another implication of the uncertainty principle is that it is impossible to accurately measure the energy of a system in some finite amount of time.

Heisenberg's uncertainty principle is a key principle in quantum mechanics. Very roughly, it states that if we know *everything* about where a particle is located (the uncertainty of position is small), we know *nothing* about its momentum (the uncertainty of momentum is large), and vice versa. Versions of the uncertainty principle also exist for other quantities as well, such as energy and time. We discuss the momentum-position and energy-time uncertainty principles separately.

Momentum and Position:

To illustrate the momentum-position uncertainty principle, consider a free particle that moves along the *x*-direction. The particle moves with a constant velocity *u* and momentum $p=mu$. According to de Broglie's relations, $p=\hbar k$ and $E=\hbar\omega$. The wave function for this particle is given by

$$\psi k(x,t)=A[\cos(\omega t-kx)-i\sin(\omega t-kx)]=Ae-i(\omega t-kx)=Ae-i\omega teikx$$

and the probability density $|\psi k(x,t)|2=A2$ is *uniform* and independent of time. The particle is equally likely to be found anywhere along the *x*-axis but has definite values of wavelength and wave number, and therefore momentum. The uncertainty of position is infinite (we are completely uncertain about position) and the uncertainty of the momentum is zero (we are completely certain about momentum). This account of a free particle is consistent with Heisenberg's uncertainty principle.

Similar statements can be made of localized particles. In quantum theory, a localized particle is modeled by a linear superposition of free-particle (or plane-wave) states called a **wave packet**. A wave packet contains many wavelengths and therefore by de Broglie's relations many momenta—possible in quantum mechanics! This particle also has many values of position, although the particle is confined mostly to the interval Δx. The particle can be better localized (Δx can be decreased) if more plane-wave states of different wavelengths or momenta are added together in the right way (Δp is increased). According to Heisenberg, these uncertainties obey the following relation.

The product of the uncertainty in position of a particle and the uncertainty in its momentum can never be less than one-half of the reduced Planck constant:

$$\Delta x \, \Delta p \geq \hbar/2.$$

This relation expresses Heisenberg's uncertainty principle. It places limits on what we can know about a particle from simultaneous measurements of position and momentum. If is large, is small, and vice versa. It can be derived in a more advanced course in modern physics. Reflecting on this relation in his work *The Physical Principles of the Quantum Theory*, Heisenberg wrote "Any use of the words 'position' and 'velocity' with accuracy exceeding that given by [the relation] is just as meaningless as the use of words whose sense is not defined."

Uncertainty principle has nothing to do with the precision of an experimental apparatus. Even for perfect measuring devices, these uncertainties would remain because they originate in the wave-like nature of matter. The precise value of the product depends on the specific form of the wave function. Interestingly, the Gaussian function (or bell-curve distribution) gives the minimum value of the uncertainty product:

Why is it Impossible to Measure both Position and Momentum Simultaneously?

In order to illustrate Heisenberg's uncertainty principle, consider an example where the position of an electron is measured. In order to measure the position of an object, a photon must collide with it and return to the measuring device. Since photons hold some finite momentum, a transfer of momenta will occur when the photon collides with the electron. This transfer of momenta will cause the momentum of the electron to increase. Thus, any attempt at measuring the position of a particle will increase the uncertainty in the value of its momentum.

Applying the same example to a macroscopic object (say a basketball), it can be understood that Heisenberg's uncertainty principle has a negligible impact on measurements in the macroscopic world. While measuring the position of a basketball, there will still be a transfer of momentum from the photons to the ball. However, the mass of the photon is much smaller than the mass of the ball. Therefore, any momentum imparted by the photon to the ball can be neglected.

Example:

The Uncertainty Principle Large and Small Determine the minimum uncertainties in the positions of the following objects if their speeds are known with a precision of 1.0×10^{-3}m/s: (a) an electron and (b) a bowling ball of mass 6.0 kg.

Strategy: Given the uncertainty in speed $\Delta u=1.0\times10^{-3}$m/s, we have to first determine the uncertainty in momentum $\Delta p=m\Delta u$ and then to find the uncertainty in position $\Delta x=\hbar/(2\Delta p)$.

Solution

a. For the electron:

$$\Delta p=m\Delta u=(9.1\times10^{-31}\text{kg})(1.0\times10^{-3}\text{m/s})=9.1\times10^{-34}\text{kg}\cdot\text{m/s},\Delta x=\frac{\hbar}{2\Delta p}=5.8\text{cm}.$$

b. For the bowling ball:

$$\Delta p=m\Delta u=(6.0\text{kg})(1.0\times10^{-3}\text{m/s})=6.0\times10^{-3}\text{kg}\cdot\text{m/s},\Delta x=\frac{\hbar}{2\Delta p}=8.8\times10^{-33}\text{m}.$$

Significance: Unlike the position uncertainty for the electron, the position uncertainty for the bowling ball is immeasurably small. Planck’s constant is very small, so the limitations imposed by the uncertainty principle are not noticeable in macroscopic systems such as a bowling ball.